The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Final Exam

Date: December 11, 2023 Course: EE 313 Evans

Name: ____
Last, First

- This in-person exam is scheduled to last two hours.
- Open books, open notes, and open class materials, including homework assignments and solution sets and previous midterm exams and solutions.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please mute all computer systems.
- Please turn off all phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote information from a source, please give the quote, page number and source citation**.**

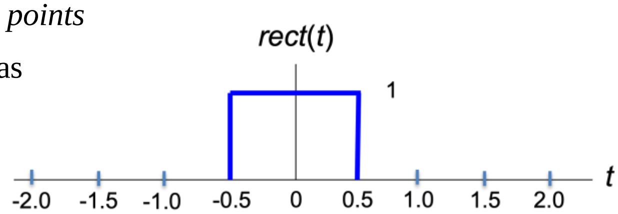
Problem	Point Value	Your Score	Topic
1	16		Continuous-Time Rectangular Pulse
2	18		Discrete-Time Convolution
3	18		Continuous-Time System Properties
4	18		Discrete-Time Filter Design
5	16		Continuous-Time Downconversion
6	14		Discrete-Time Mystery Systems
Total	100		

Problem 1. Continuous-Time Rectangular Pulse. *16 points*

Consider a rectangular pulse signal $rect(t)$ defined as

$$rect(t) = \begin{cases} 1 & \text{for } -0.5 \le t < 0.5 \\ 0 & \text{otherwise} \end{cases}$$

which is plotted on the right.



(a) Plot $rect(t - 0.5)$. *4 points*.

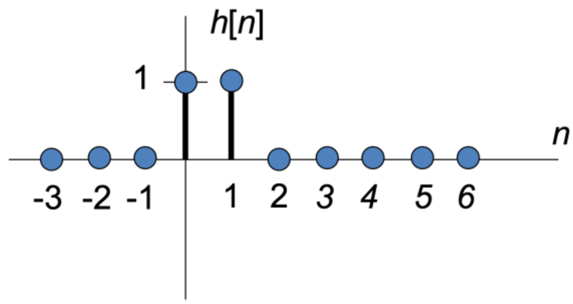
(b) Plot $rect(0.5 - t)$. *4 points*.

(c) Plot $rect\left(\frac{t}{2}\right)$. *4 points*.

(d) Plot $rect\left(\frac{t-0.5}{2}\right)$. *4 points*.

Problem 2. Discrete-Time Convolution. *18 points*

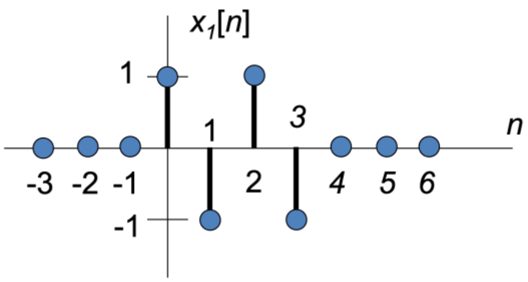
Consider a discrete-time linear time-invariant (LTI) system with impulse response $h[n] = \delta[n] + \delta[n-1]$ plotted on the right.



For each of the following input signals,
i. give a formula for the input signal. *2 points each.*
ii. plot the output signal $y[n]$. *4 points each.*

(a) $x_1[n] =$ ______________________________

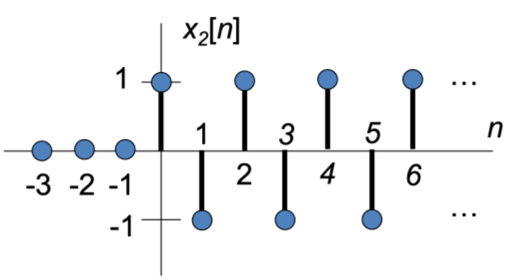
Here, $x_1[n]$ has four non-zero values.



Plot $y[n] = h[n] * x_1[n]$

(b) $x_2[n] =$ ______________________________

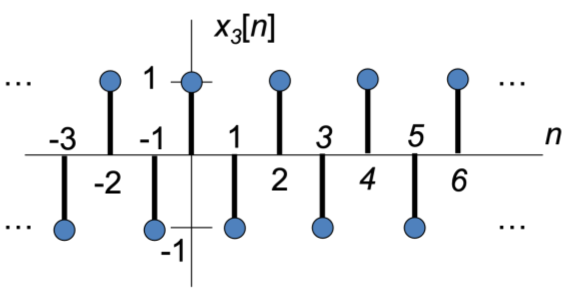
Here, $x_2[n]$ is 0 for $n < 0$. For $n \geq 0$, $x_2[n]$ alternates between 1 and -1 indefinitely.



Plot $y[n] = h[n] * x_2[n]$

(c) $x_3[n] =$ ______________________________

Here, $x_3[n]$ alternates between 1 and -1 for all n.



Plot $y[n] = h[n] * x_3[n]$

Problem 3. Continuous-Time System Properties. *18 points*

Each continuous-time system has input $x(t)$ and output $y(t)$, and $x(t)$ and $y(t)$ might be complex-valued.

Determine if each system is linear or nonlinear, time-invariant or time-varying, and bounded-input bounded-output (BIBO) stable or unstable.

You must either prove that the system property holds in the case of linearity, time-invariance, or stability, or provide a counter-example that the property does not hold. Providing an answer without any justification will earn 0 points.

Part	***System Name***	***System Formula***	***Linear?***	***Time-Invariant?***	***BIBO Stable?***
(a)	Integrator	$y(t) = C_0 + \int_{0^-}^{t} x(u)\, du$ for $t \geq 0^-$ and $C_0 = 5$			
(b)	Amplitude Modulation	$y(t) = x(t) \cos(2\, \pi\, f_c\, t)$ for $t \geq 0$ where f_c is a constant			
(c)	Reciprocal	$y(t) = \frac{1}{x(t)}$ for $-\infty < t < \infty$			

(a) Integrator: $y(t) = C_0 + \int_{0^-}^{t} x(u)\, du$ for $t \geq 0^-$ and $C_0 = 5$. *6 points.*

(b) Amplitude Modulation: $y(t) = x(t) \cos(2\, \pi\, f_c\, t)$ for $t \geq 0$ where f_c is a constant. *6 points.*

(c) Reciprocal: $y(t) = \frac{1}{x(t)}$ for $-\infty < t < \infty$. *6 points.*

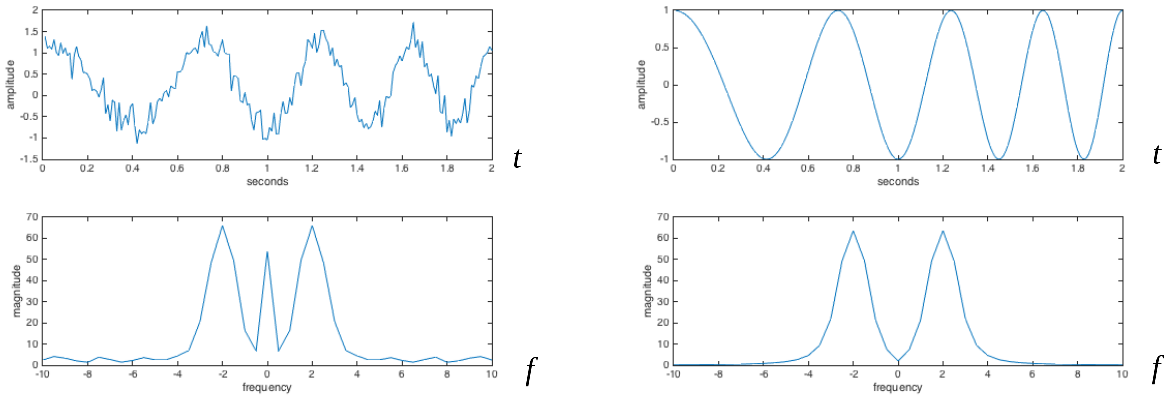
Problem 4. Discrete-Time Filter Design. *18 points*.

A sinusoidal signal of interest has a principal frequency that can vary over time in the range 1-3 Hz.

Using a sampling rate of f_s = 20 Hz, a sinusoidal signal was acquired for 2s and shown below on the left in the upper plot. The lower plot is the magnitude of the signal's frequency content.

The acquired signal has interference and other impairments that reduce the signal quality.

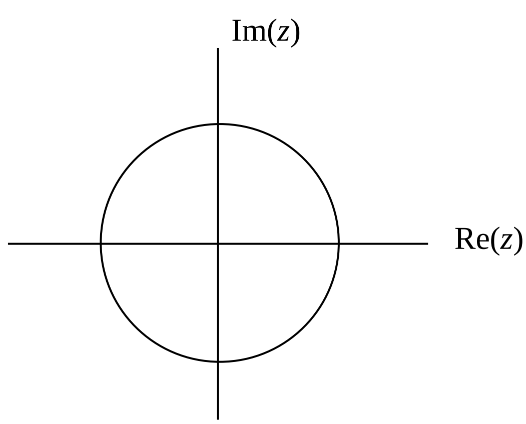
The signal shown below on the right is the sinusoidal signal without the impairments.



Design a second-order infinite impulse response (IIR) filter to filter the acquired signal above on the left to give the sinusoidal signal above on the right. Filter should be bounded-input bounded-output stable.

(a) Give the poles and zeros of the second-order IIR filter. Explain why you chose the poles and zeros. *12 points*.

(b) Draw the pole-zero diagram for the second-order IIR filter. *6 points*.



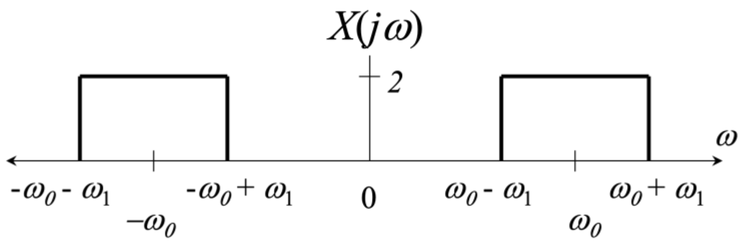
Problem 5. Continuous-Time Downconversion. *16 points*.

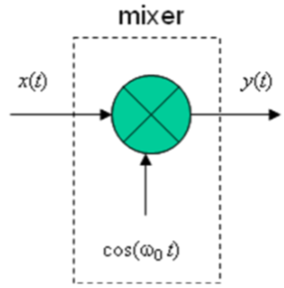
A signal $x(t)$ is input to a mixer to produce the output $y(t)$ where

$$y(t) = x(t)\cos(\omega_0 t)$$

where $\omega_0 = 2\pi f_0$ and $f_0 = 5$ kHz. A block diagram of the mixer is shown below on the right.

The Fourier transform of $x(t)$ is shown below on the left where $\omega_1 = 2\pi f_1$ and $f_1 = 1$ kHz.





(a) Using Fourier transform properties, derive an expression for $Y(j\omega)$ in terms of $X(j\omega)$. *8 points.*

(b) Sketch $Y(j\omega)$ vs. ω. Label all important points on the horizontal and vertical axes. *6 points.*

(c) What operation would you apply to the signal $y(t)$ in part (b) to pass frequencies from $-\omega_1$ to ω_1 and attenuate other frequencies? This will allow us to recover the message signal that was transmitted using amplitude modulation. *2 points.*

Problem 6. Discrete-Time Mystery Systems. *14 points*.

You're trying to identify unknown discrete-time systems.

You input a discrete-time chirp signal $x[n]$ and look at the output to figure out what the system is.

The discrete-time chirp is formed by sampling a chirp signal that sweeps 0 to 8000 Hz over 0 to 5s

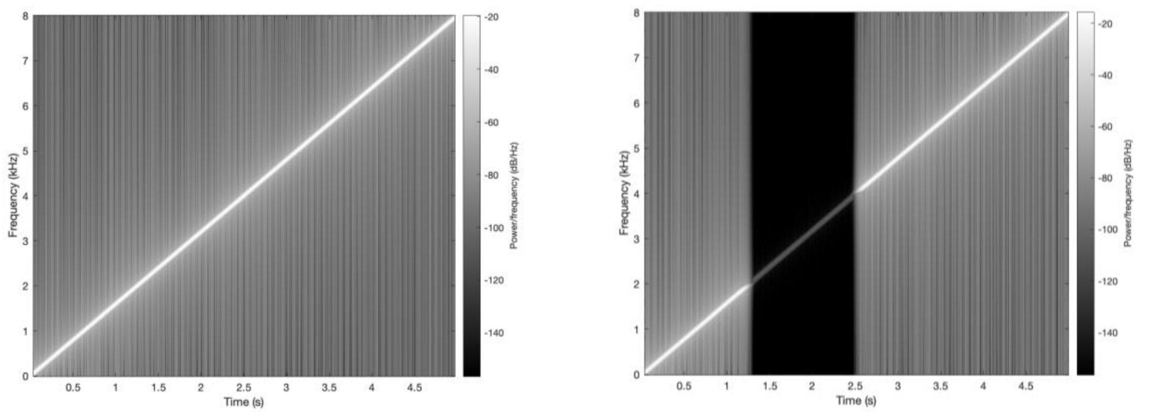
$$x(t) = \cos(2\pi f_1 t + 2\pi \mu t^2)$$

where $f_1 = 0$ Hz, $f_2 = 8000$ Hz, and $\mu = \frac{f_2 - f_1}{2\, t_{\max}} = \frac{8000 \text{ Hz}}{10\, s} = 800 \text{ Hz}^2$. Sampling rate f_s is 16000 Hz.

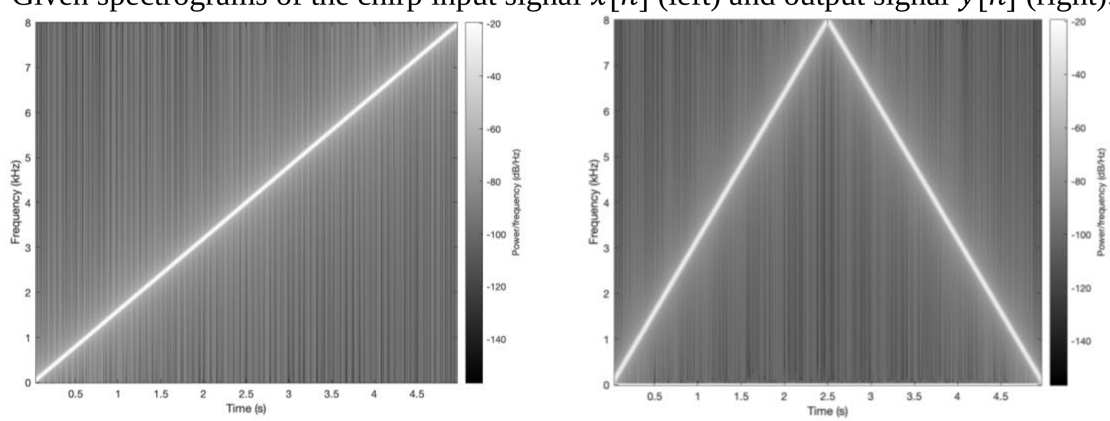
In part (a) and (b) blow, identify the unknown system as one of the following **with justification**:

1. filter – give selectivity (lowpass, highpass, bandpass, bandstop) and passband/stopband frequencies
2. pointwise nonlinearity – give the integer exponent k to produce the output $y[n] = x^k[n]$

(a) Given spectrograms of the chirp input signal $x[n]$ (left) and output signal $y[n]$ (right). *7 points*.



(b) Given spectrograms of the chirp input signal $x[n]$ (left) and output signal $y[n]$ (right). *7 points.*



Please note that the output spectrogram has a strong component at DC (0 rad/sample).